

Nilpotent Symmetries of a Diffeomorphism Invariant Theory: BRST Approach

R. P. Malik^(a,b)

^(a) *Physics Department, Institute of Science,
Banaras Hindu University, Varanasi - 221 005, (U.P.), India*

^(b) *DST Centre for Interdisciplinary Mathematical Sciences,
Institute of Science, Banaras Hindu University, Varanasi - 221 005, India*
e-mail: rpmalik1995@gmail.com

Abstract: Within the framework of Becchi-Rouet-Stora-Tyutin (BRST) formalism, we discuss the *full* set of proper BRST and anti-BRST transformations for a diffeomorphism invariant theory which is described by the Lagrangian density of a standard bosonic string (proposed by Kato and Ogawa). The above (anti-)BRST symmetry transformations are off-shell nilpotent and absolutely anticommuting. The latter property is valid on a constrained hypersurface in the two dimensional spacetime manifold (traced out by the propagation of the bosonic string) where the Curci-Ferrari (CF) type restriction is satisfied. This CF-type restriction is found to be an (anti-)BRST invariant quantity. We derive the precise form of the BRST and anti-BRST invariant Lagrangian densities as well as the exact expressions for the conserved (anti-)BRST and ghost charges of our present theory. The derivation of the *proper* anti-BRST symmetry transformations and the emergence of the CF-type restriction are completely *novel* results in our present investigation.

PACS numbers: 04.60.Cf., 11.25.Sq, 11.30.-j

Keywords: Diffeomorphism invariance, bosonic string theory, (anti-)BRST symmetries, nilpotency, absolute anticommutativity, CF-type restrictions

1 Introduction

One of the most exciting and captivating areas of research in theoretical high energy physics (THEP), over the last few decades, has been the subject of (super)strings and related extended objects (see, e.g. [1-4] for details). This is due to the fact that, in one stroke, these theories provide a possible scenario of unification of *all* the fundamental interactions of nature and a promising candidate for the precise theory of quantum gravity. The modern developments in the realm of (super)strings have influenced many other areas of research in THEP, e.g. non-commutative field theories, higher p -form ($p = 2, 3, 4, \dots$) gauge theories, higher spin gauge theories, supersymmetric gauge theories and related mathematics, gauge-gravity duality, AdS/CFT correspondence, etc. The quantization of these (super)string theories have led us to imagine about the higher dimensional view of the physical world we live in. It has been established that one cannot consistently quantize the dual-string theory [5] unless the spacetime dimension $D = 26$ and the intercept of the leading Regge trajectory $\alpha_0 = 1$. These results have been obtained and formally established from many different considerations like the requirement of the validity of proper Lorentz algebra, unitarity requirements of these string theories, nilpotency of the Becchi-Rouet-Stora-Tyutin (BRST) charge, etc. In this context one of the earliest attempts to covariantly quantize a bosonic string theory, within the framework of BRST formalism, was undertaken by Kato and Ogawa [6] where the diffeomorphism symmetry of this theory was exploited.

In the above work [6], it is precisely the infinitesimal version of the diffeomorphism symmetry invariance of the theory that has been primarily exploited to perform the BRST quantization where *only* the BRST symmetries have been discussed. However, there is no discussion about the anti-BRST symmetries and related Curci-Ferrari (CF)-type restrictions which are the hallmarks of a proper BRST quantization scheme. In this work [6], the metric tensor has been taken in such a manner that the conformal anomaly does not spoil the BRST analysis. In fact, the metric tensor has been decomposed in such a way that it has *three* independent degrees of freedom to begin with. A Lagrange multiplier field (density) has been introduced so that the equation of motion w.r.t. *it* puts a restriction on the determinant of the metric tensor. The latter condition reduces the independent degrees of freedom of the metric tensor from *three* to *two*. The BRST charge has been calculated in the flat limit where the metric tensor becomes Minkowskian in nature (see, e.g. [6] for more details). The nilpotency requirement of this BRST charge leads to the derivation of $D = 26$ and $\alpha_0 = 1$. One of the central theme of our present investigation is to focus on the existence of (i) the proper anti-BRST symmetries (corresponding to the BRST transformations taken in [6]), and (ii) the (anti-)BRST invariant CF-type of restrictions. We have taken a modest step in this direction in our present endeavor.

We have performed the full BRST analysis of the above theory [6] in the sense that we have derived the *proper* anti-BRST symmetry transformations corresponding to the BRST symmetry transformations that have been taken into account in [6]. The BRST and anti-BRST symmetry transformations are found to be off-shell nilpotent and absolutely anticommuting in nature. The latter property has been shown to be true on a hypersurface where the CF-type restrictions (11) are satisfied (see below). We observe that these restrictions are BRST as well as anti-BRST invariant thereby implying that these are *physical* restrictions (which can be imposed from *outside* on our present theory). We have derived,

in our present endeavor, the BRST and anti-BRST invariant Lagrangian densities and have shown explicitly their BRST and anti-BRST invariance. The conserved charges of the theory have been computed in the flat limit where $A_0 = A_1 = 0, A_2 = 1$. In fact, the latter conditions imply that the metric tensor of the theory transforms as: $\tilde{g}^{ab} \rightarrow \eta^{ab}$ where η^{ab} is the flat metric of the 2D Minkowski space (which is nothing but the 2D surface traced out by the propagation of the bosonic string). We have also established that the standard algebra between the ghost charge and BRST charge (as well as between the ghost charge and anti-BRST charge) is satisfied. We have also commented, very briefly, about the nilpotency properties of the BRST and anti-BRST charges which are true at the *quantum* level only when $D = 26$ and $\alpha_0 = 1$ provided we take into account the normal mode expansions of the fields (consistent with the appropriate boundary conditions) and substitute them in the computation of $Q_B^2 = \frac{1}{2} \{Q_B, Q_B\} = 0$ and $\bar{Q}_B^2 = \frac{1}{2} \{\bar{Q}_B, \bar{Q}_B\} = 0$.

The main motivating factors behind our present investigation are as follows. First, in the BRST description [6] of the present bosonic string, *only* the BRST symmetry transformations have been discussed corresponding to the infinitesimal diffeomorphism invariance of the theory. The nilpotent anti-BRST symmetry transformations have remained untouched in [6]. Thus, it is important for us to discuss the BRST as well as anti-BRST symmetry transformations *together* for the *complete* BRST analysis of our present theory. We have accomplished this goal in our present endeavor. Second, in the BRST description of Kato and Ogawa [6], the auxiliary fields have been modified/redefined in a very complicated fashion to simplify the theoretical analysis of the present theory. There is, however, no basic physical arguments to support such kind of modifications/redefinitions. We have, in our present endeavor, *not* invoked any such kind of modifications/redefinitions as our analysis is very straightforward. Third, the hallmark of a quantum theory, discussed within the framework of BRST formalism, is the existence of the (non-)trivial Curci-Ferrari (CF) type restriction(s). We have derived such a restriction in our present endeavor which ensures the absolute anticommutativity of the (anti-)BRST symmetry transformations. Finally, our present work is important because, for this model, the recently developed superfield approach [7] would be very useful because our theory is diffeomorphism invariant. We hope that the application of this superfield formalism [7] would shed some *new* lights on the specific aspects of our present theory (as far as the symmetries are concerned).

Our present paper is organized as follows. To set up the notations and convention, we discuss very briefly the diffeomorphism symmetry as well as the corresponding BRST approach in Sec. 2 which has been performed in [6]. Our Sec. 3 is devoted to the discussion of BRST and anti-BRST symmetries where we also point out the existence of the CF-type restriction. We prove the (anti-)BRST invariance of this restriction as well as we demonstrate the nilpotency as well as the absolute anticommutativity of the (anti-)BRST symmetry transformations. We derive the explicit form of the BRST as well as anti-BRST invariant Lagrangian densities in Sec. 4. The conserved charges, corresponding to the continuous *internal* symmetries of the theory, are derived in Sec. 5. in the flat limit. Finally, we make some concluding remarks on our present investigation in Sec. 6 and point out a few future directions for further investigations.

In our Appendices A, B and C, we incorporate some of the algebraic expressions (as well as equations) that have been used in the main body of our text.

2 Preliminary: Diffeomorphism and BRST Invariance

We begin with the Lagrangian density of a bosonic string theory as (see, e.g. [6] for details)

$$\mathcal{L}_0 = -\frac{1}{2k} \tilde{g}^{ab} \partial_a X^\mu \partial_b X_\mu + E (\det \tilde{g} + 1), \quad (1)$$

where $\tilde{g}^{ab} = \sqrt{-g} g^{ab}$ has two independent degrees of freedom* because $\det \tilde{g} = -1$ due to the equation of motion w.r.t. the Lagrange multiplier field E which happens to be a scalar density (cf. Eqn. (2) below). Here the 2D surface, traced out by the propagation of the bosonic string, is parameterized by $\xi^a = (\xi^0, \xi^1) \equiv (\tau, \sigma)$ where $a = 0, 1$ and component parameters (τ, σ) satisfy: $-\infty < \tau < +\infty$ and $0 \leq \sigma \leq \pi$. The string coordinates $X^\mu(\xi)$ (with $\mu = 0, 1, 2, 3, \dots, D-1$) are in the D-dimensional flat Minkowskian spacetime manifold and $\tilde{g}^{ab} = \sqrt{-g} g^{ab}$ is the metric tensor constructed with the determinant ($g = \det g_{ab}$) and inverse (g^{ab}) of the metric tensor g_{ab} of the 2D parameter space. Under the infinitesimal diffeomorphism transformations[†]: $\xi^a \rightarrow \xi^a - \varepsilon^a(\xi)$, we have the following transformations (δ_ε) on the relevant fields of our present bosonic string theory[‡], namely;

$$\begin{aligned} \delta_\varepsilon X^\mu &= \varepsilon^a \partial_a X^\mu, & \delta_\varepsilon E &= \partial_a (\varepsilon^a E), & \delta_\varepsilon (\det \tilde{g}) &= \varepsilon^a \partial_a (\det \tilde{g}), \\ \delta_\varepsilon \tilde{g}^{ab} &= \partial_m (\varepsilon^m \tilde{g}^{ab}) - (\partial_m \varepsilon^a) \tilde{g}^{mb} - (\partial_m \varepsilon^b) \tilde{g}^{am}, \end{aligned} \quad (2)$$

where $\varepsilon^a(\xi)$ are the infinitesimal diffeomorphism transformation parameters. The above transformations leave the Lagrangian density (1) quasi-invariant (i.e. $\delta_\varepsilon \mathcal{L}_0 = \partial_a (\varepsilon^a \mathcal{L}_0)$). This demonstrates that the action integral $S = \int d^2 \xi \mathcal{L}_0 \equiv \int_{-\infty}^{+\infty} d\tau \int_0^\pi d\sigma \mathcal{L}_0$ remains invariant under the diffeomorphism transformations (2) provided the boundary conditions: $\varepsilon^a(\xi) = 0$ at $\sigma = 0$ and $\sigma = \pi$ are imposed on the diffeomorphism parameter $\varepsilon^a(\xi)$.

For the BRST quantization of the Lagrangian density (1), we have to invoke the gauge-fixing conditions. This can be achieved if we take the following decomposition for the metric tensor \tilde{g}^{ab} (see, e.g. [6] for details)

$$\tilde{g}^{ab} = \begin{pmatrix} A_1 + A_2 & A_0 \\ A_0 & A_1 - A_2 \end{pmatrix}, \quad (3)$$

*The original Lagrangian density: $-\frac{1}{2k} \sqrt{-g} g^{ab} \partial_a X^\mu \partial_b X_\mu$ is endowed with the local conformal invariance. However, this conformal invariance is broken by the conformal anomaly [8,9] if we regularize the system in a gauge-invariant manner. We have avoided this problem by taking $\tilde{g}^{ab} = \sqrt{-g} g^{ab}$ as the metric tensor of our present theory [6] with *three* independent degrees of freedom to start with. The EoM w.r.t. E (i.e. $\det \tilde{g} = -1$) reduces the independent degree of freedom to *two*.

[†]It will be noted that we differ from [6] by an overall sign factor in the diffeomorphism transformations (2) and BRST transformations (6) because we have chosen the infinitesimal diffeomorphism transformation $\xi^a \rightarrow \xi^a - \varepsilon^a(\xi)$ whereas the same transformation has been taken as: $\xi^a \rightarrow \xi^a + \varepsilon^a(\xi)$ in [6].

[‡]We choose the Latin indices $a, b, c, \dots, l, m, n, \dots = 0, 1$ to denote τ and σ directions on the 2D surface (traced out by the propagation of the bosonic string) and the Greek indices $\mu, \nu, \lambda, \dots = 0, 1, 2, \dots, D-1$ stand for the spacetime directions of the D-dimensional flat Minkowskian spacetime manifold corresponding to the target space. The above 2D surface is embedded in the D-dimensional Minkowskian flat target space (which turns out to be 26 at the *quantum* level). Throughout the whole body of our text, we denote the BRST and anti-BRST symmetry transformations by the symbols s_B and \bar{s}_B , respectively. We adopt the convention of left-derivative w.r.t. the fermionic fields (C^a, \bar{C}^a , etc.) of our present theory. Consistent with this convention, the Noether conserved currents in equations (25) and (26) are defined (see below).

and set the gauge-fixing conditions $A_0 = A_1 = 0$ so that we obtain $\det \tilde{g} = -A_2^2 = -1$. This shows that, for the choice $A_2 = 1$, we obtain the flatness condition[§] $\tilde{g}^{ab} \rightarrow \eta^{ab}$ with the signatures $(+1, -1)$. By exploiting the standard techniques of the BRST formalism [10,11], we obtain the gauge-fixing and Faddeev-Popov ghost terms for the theory, in the language of the nilpotent ($s_B^2 = 0$) BRST transformations s_B , as (see, e.g. [10,11] for details)

$$\mathcal{L}_{GF} + \mathcal{L}_{FP} = s_B \left[-i \bar{C}_0 A_0 - i \bar{C}_1 A_1 \right], \quad (4)$$

where \bar{C}_0 and \bar{C}_1 are the anti-ghost fields with ghost number (-1) . It will be noted that the transformations $s_B \bar{C}_0 = i B_0$ and $s_B \bar{C}_1 = i B_1$ lead to the emergence of the Nakanishi-Lautrup type auxiliary fields of the theory as B_0 and B_1 and the nilpotency requirements produce $s_B B_0 = s_B B_1 = 0$. A close look at the transformations (2) and decomposition (3) leads to the following BRST symmetry transformations for the component gauge fields

$$\begin{aligned} s_B A_0 &= C^a \partial_a A_0 - (\partial_0 C^1 + \partial_1 C^0) A_1 - (\partial_0 C^1 - \partial_1 C^0) A_2, \\ s_B A_1 &= C^a \partial_a A_1 - (\partial_0 C^0 - \partial_1 C^1) A_2 - (\partial_1 C^0 + \partial_0 C^1) A_0, \\ s_B A_2 &= C^a \partial_a A_2 - (\partial_0 C^0 - \partial_1 C^1) A_1 - (\partial_1 C^0 - \partial_0 C^1) A_0, \end{aligned} \quad (5)$$

where we have taken the replacement ($\varepsilon^a \rightarrow C^a$) which implies that the infinitesimal diffeomorphism parameters (ε^a , $a = 0, 1$) have been replaced by the fermionic ($(C^a)^2 = 0$, $C^a C^b + C^b C^a = 0$) ghost fields C^a . As a consequence of this replacement, we have the following BRST symmetry transformations *vis-à-vis* the transformations (2), namely;

$$\begin{aligned} s_B X^\mu &= C^a \partial_a X^\mu, & s_B E &= \partial_a (C^a E), & s_B (\det \tilde{g}) &= \varepsilon^a \partial_a (\det \tilde{g}), \\ s_B C^a &= C^b \partial_b C^a, & s_B \bar{C}^a &= i B^a, & s_B B^a &= 0, \\ s_B \tilde{g}^{ab} &= \partial_m (C^m \tilde{g}^{ab}) - (\partial_m C^a) \tilde{g}^{mb} - (\partial_m C^b) \tilde{g}^{am}, \end{aligned} \quad (6)$$

where the transformation $s_B C^a = C^b \partial_b C^a$ has been derived from the requirement of nilpotency condition ($s_B^2 X^\mu = 0$). With the inputs from (5) and (6), we obtain the BRST invariant Lagrangian density (\mathcal{L}_B) from (4) and (1), modulo some total derivatives, as[¶]:

$$\begin{aligned} \mathcal{L}_B &= \mathcal{L}_0 + B_0 A_0 + B_1 A_1 + i \left[C^a \partial_a \bar{C}_0 - \bar{C}_0 (\partial_a C^a) - \bar{C}_1 (\partial_0 C^1 + \partial_1 C^0) \right] A_0 \\ &+ i \left[C^a \partial_a \bar{C}_1 - \bar{C}_1 (\partial_a C^a) - \bar{C}_0 (\partial_0 C^1 + \partial_1 C^0) \right] A_1 \\ &+ i \left[\bar{C}_0 (\partial_0 C^1 - \partial_1 C^0) + \bar{C}_1 (\partial_0 C^0 - \partial_1 C^1) \right] A_2. \end{aligned} \quad (7)$$

The above full Lagrangian density, under the flatness limit $A_0 = A_1 = 0, A_2 = 1$, reduces to the following Lagrangian density^{||}

$$\begin{aligned} \mathcal{L}_B \longrightarrow \mathcal{L}_B^{(0)} &= -\frac{1}{2\kappa} \eta^{ab} \partial_a X^\mu \partial_b X_\mu + E (1 - A_2^2) + B_0 A_0 + B_1 A_1 \\ &+ i \left[\bar{C}_0 (\partial_0 C^1 - \partial_1 C^0) + \bar{C}_1 (\partial_0 C^0 - \partial_1 C^1) \right], \end{aligned} \quad (8)$$

[§]We shall take the *flatness condition* $\tilde{g}^{ab} \rightarrow \eta^{ab}$ in the language of restrictions on the component gauge fields: $A_0 = A_1 = 0, A_2 = 1$ for the full discussion of our theory within the framework of BRST formalism.

[¶]The Lagrangian density \mathcal{L}_B has been written, modulo some total derivatives, in such a manner that BRST transformations (5) could be implemented in a simple and straightforward manner.

^{||}We would like to point out that the flatness limit (i.e. $A_0 = A_1 = 0, A_2 = 1$) has been taken in all the terms of (7) except the gauge-fixing terms (i.e. $B_0 A_0 + B_1 A_1$) and the Lagrange multiplier term (i.e. $E (1 - A_2^2)$) because the EoM w.r.t. B_0, B_1 and E imply the same thing (i.e. $A_0 = A_1 = 0, A_2 = 1$).

which has been obtained in [6] after taking the help of the redefinitions of the auxiliary fields in a complicated fashion. In fact, these redefinitions are mathematical in nature and there is *no* physical arguments to support the specific choices that have been made in [6] for the simplification of the Lagrangian density in the flat space. We have obtained (8) from (7) in a straightforward manner (without any redefinitions/modifications, etc.).

3 BRST and Anti-BRST Symmetries: Key Features

It can be checked, in a straightforward fashion, that the BRST symmetry transformations, quoted in (5) and (6), are nilpotent of order two (i.e. $s_B^2 = 0$). The proper anti-BRST symmetry transformations, corresponding to the BRST transformations (6), are

$$\begin{aligned}\bar{s}_B X^\mu &= \bar{C}^a \partial_a X^\mu, & \bar{s}_B \bar{C}^a &= \bar{C}^b \partial_b \bar{C}^a, & \bar{s}_B C^a &= i \bar{B}^a, \\ \bar{s}_B E &= \partial_a (\bar{C}^a E), & \bar{s}_B (\det \tilde{g}) &= \bar{C}^a \partial_a (\det \tilde{g}), & \bar{s}_B \bar{B}^a &= 0, \\ \bar{s}_B \tilde{g}^{ab} &= \partial_m (\bar{C}^m \tilde{g}^{ab}) - (\partial_m \bar{C}^a) \tilde{g}^{mb} - (\partial_m \bar{C}^b) \tilde{g}^{am},\end{aligned}\tag{9}$$

which are off-shell nilpotent ($\bar{s}_B^2 = 0$). It will be noted that we have invoked a new Nakanishi-Lautrup type of auxiliary field $\bar{B}^a(\xi)$ in our theory. Thus, we observe that the symmetry transformations (9), (6) and (5) satisfy *one* (i.e. nilpotency) of the two sacrosanct properties (i.e. nilpotency and absolute anticommutativity) that have to be satisfied by any *proper* (anti-)BRST symmetry transformations. We further note that the last entry of (9) can be written in terms of A_0, A_1, A_2 in the following form, namely;

$$\begin{aligned}\bar{s}_B A_0 &= \bar{C}^a \partial_a A_0 - (\partial_0 \bar{C}^1 + \partial_1 \bar{C}^0) A_1 - (\partial_0 \bar{C}^1 - \partial_1 \bar{C}^0) A_2, \\ \bar{s}_B A_1 &= \bar{C}^a \partial_a A_1 - (\partial_0 \bar{C}^0 - \partial_1 \bar{C}^1) A_2 - (\partial_1 \bar{C}^0 + \partial_0 \bar{C}^1) A_0, \\ \bar{s}_B A_2 &= \bar{C}^a \partial_a A_2 - (\partial_0 \bar{C}^0 - \partial_1 \bar{C}^1) A_1 - (\partial_1 \bar{C}^0 - \partial_0 \bar{C}^1) A_0.\end{aligned}\tag{10}$$

Thus, it is clear that the anti-BRST transformations for A_0, A_1, A_2 are exactly same as equation (5) with the replacement: $C^a \rightarrow \bar{C}^a$.

We dwell a bit now on the absolute anticommutativity property (i.e. $\{s_B, \bar{s}_B\} = 0$) of the (anti-)BRST symmetry transformations (10), (9), (6) and (5). It turns out that the requirement of $\{s_B, \bar{s}_B\} X^\mu = 0$ leads to the existence of the following Curci-Ferrari (CF)-type restrictions (which are primarily *two* in numbers), namely;

$$B^a + \bar{B}^a + i (C^b \partial_b \bar{C}^a + \bar{C}^b \partial_b C^a) = 0, \quad (a = 0, 1).\tag{11}$$

It turns out that the above conditions (11) have to be imposed to obtain the absolute anticommutativity (i.e. $\{s_B, \bar{s}_B\} = 0$) property when *all* the relevant fields of the whole theory are taken into account. For instance, it can be checked that the requirement of $\{s_B, \bar{s}_B\} E = 0$ *also* requires the validity of the CF-type restrictions (11). Furthermore, we obtain the following (anti-)BRST symmetry transformations on the Nakanishi-Lautrup auxiliary fields $B^a(\xi)$ and $\bar{B}^a(\xi)$ due to the requirement of the absolute anticommutativity property (e.g. $\{s_B, \bar{s}_B\} C^a = 0$ and $\{s_B, \bar{s}_B\} \bar{C}^a = 0$), namely;

$$s_B \bar{B}^a = C^b \partial_b \bar{B}^a - \bar{B}^b \partial_b C^a, \quad \bar{s}_B B^a = \bar{C}^b \partial_b B^a - B^b \partial_b \bar{C}^a.\tag{12}$$

Interestingly, the above transformations *also* satisfy the off-shell nilpotency property (i.e. $s_B^2 = 0$, $\bar{s}_B^2 = 0$) which is one of the key requirements of a proper set of (anti-)BRST symmetry transformations. Thus, we note that the (anti-)BRST symmetry transformations (12), (10), (9), (6) and (5) satisfy the off-shell nilpotency and absolute anticommutativity on a constrained hypersurface in the 2D space where the CF-restrictions (11) are satisfied.

We would enumerate here some of the subtle features associated with the CF-type restrictions (11) which are at the heart of the absolute anticommutativity property of our BRST and anti-BRST symmetry transformations. First, we note that this restriction is (anti-)BRST invariant quantity, namely;

$$\begin{aligned}\bar{s}_B \left[B^a + \bar{B}^a + i \left(C^b \partial_b \bar{C}^a + \bar{C}^b \partial_b C^a \right) \right] &= 0, \\ s_B \left[B^a + \bar{B}^a + i \left(C^b \partial_b \bar{C}^a + \bar{C}^b \partial_b C^a \right) \right] &= 0.\end{aligned}\tag{13}$$

This demonstrates that the CF-type restrictions of our present theory are *physical* (in some sense) and the hypersurface defined by it is physically relevant. This demonstrates that our (anti-)BRST invariant theory is *consistently* defined on a hypersurface where the CF-type restrictions (11) are always valid. In fact, on this hypersurface *alone*, the BRST and anti-BRST symmetry transformations have their own identities as they are linearly independent of each-other (due to their absolute anticommutativity). In the proof of (13), it is obvious that we have taken into account the (anti-)BRST transformations (12), (9) and (6).

We end this section with the following remarks on the nilpotency properties (i.e. $s_B^2 = 0$, $\bar{s}_B^2 = 0$) associated with the BRST and anti-BRST symmetry transformations s_B and \bar{s}_B . First of all, we note that $s_B^2 X^\mu = 0$ leads to the derivation of $s_B C^a = C^b \partial_b C^a$ where $s_B^2 C^a = 0$ is *also* satisfied. In exactly similar fashion, we obtain $\bar{s}_B \bar{C}^a = \bar{C}^b \partial_b \bar{C}^a$ (where $\bar{s}_B^2 \bar{C}^a = 0$) from the requirement of nilpotency of the anti-BRST symmetry transformation on X^μ field ($\bar{s}_B^2 X^\mu = 0$). The proof of the nilpotency (i.e. $s_B^2 = 0$, $\bar{s}_B^2 = 0$) of the transformations $s_B \tilde{g}^{ab}$ and $\bar{s}_B \tilde{g}^{ab}$ (cf. Eqns. (6) and (9)) is algebraically more involved**. However, it turns out that $s_B^2 \tilde{g}^{ab} = 0$ and $\bar{s}_B^2 \tilde{g}^{ab} = 0$ are *indeed* true. This proof, in turn, implies that the (anti-)BRST transformations (cf. Eqns. (5) and (10)) of the component gauge fields (i.e. A_0, A_1, A_2) are *automatically* nilpotent (i.e. $s_B^2 = 0$, $\bar{s}_B^2 = 0$) of order two.

4 (Anti-)BRST Invariant Lagrangian Densities

We have already mentioned the BRST invariant Lagrangian densities (7) and (8) in the (non-)flat limits. In our present section, we shall establish their BRST invariance. The analogue of the Lagrangian density (7) that remains invariant, under the anti-BRST symmetry transformations (9), (10) and (12), is as follows [10,11]:

$$\mathcal{L}_{\bar{B}} = \mathcal{L}_0 + \bar{s}_B \left[i C_0 A_0 + i C_1 A_1 \right].\tag{14}$$

**We have collected some of the crucial expressions (as well as equations) in our Appendix A which establish the nilpotency ($s_B^2 \tilde{g}^{ab} = 0$) of the BRST transformations when they act on the metric tensor \tilde{g}^{ab} .

Using the explicit anti-BRST symmetry transformations (9), (10) and (12), we obtain the following explicit form of the anti-BRST invariant Lagrangian density $\mathcal{L}_{\bar{B}}$ as

$$\begin{aligned}\mathcal{L}_{\bar{B}} &= \mathcal{L}_0 - \bar{B}_0 A_0 - \bar{B}_1 A_1 + i \left[C_0 (\partial_a \bar{C}^a) + (\partial_a C_0) \bar{C}^a + C_1 (\partial_0 \bar{C}^1 + \partial_1 \bar{C}^0) \right] A_0 \\ &+ i \left[C_1 (\partial_a \bar{C}^a) + (\partial_a C_1) \bar{C}^a + C_0 (\partial_0 \bar{C}^1 + \partial_1 \bar{C}^0) \right] A_1 \\ &+ i \left[C_0 (\partial_0 \bar{C}^1 - \partial_1 \bar{C}^0) + C_1 (\partial_0 \bar{C}^0 - \partial_1 \bar{C}^1) \right] A_2,\end{aligned}\tag{15}$$

where some total derivative terms have been dropped as they do not affect the dynamics of the theory. The flat limit (i.e. $A_0 = A_1 = 0$, $A_2 = 1$) of the above Lagrangian density, in its full blaze of glory, is as follows:

$$\begin{aligned}\mathcal{L}_{\bar{B}} \rightarrow \mathcal{L}_{\bar{B}}^{(0)} &= -\frac{1}{2\kappa} \eta^{ab} \partial_a X^\mu \partial_b X_\mu + E (1 - A_2^2) - \bar{B}_0 A_0 - \bar{B}_1 A_1 \\ &+ i \left[C_0 (\partial_0 \bar{C}^1 - \partial_1 \bar{C}^0) + C_1 (\partial_0 \bar{C}^0 - \partial_1 \bar{C}^1) \right],\end{aligned}\tag{16}$$

where the above limit has not been imposed on the gauge-fixing terms ($-\bar{B}_0 A_0 - \bar{B}_1 A_1$) and the term $(E (1 - A_2^2))$ with the Lagrange multiplier field. We shall be calculating the conserved charges of the theory from the Lagrangian densities (8) and (16) which are quoted in the flat limits (cf. Sec. 5 below for details).

To establish the explicit (anti-)BRST invariance of the Lagrangian densities (7), (8), (15) and (16), we have to apply the (anti-)BRST transformations on every terms of the above Lagrangian densities. This exercise is algebraically more involved as one has to collect the terms containing A_0 , A_1 , A_2 , B_0 , B_1 , separately and independently. In our Appendices B and C, we have collected these terms which appear due to the applications of s_B and \bar{s}_B on the Lagrangian densities \mathcal{L}_B and $\mathcal{L}_{\bar{B}}$, respectively. The explicit form of the BRST transformations on the BRST invariant Lagrangian density (7) (i.e. \mathcal{L}_B) is

$$\begin{aligned}s_B \mathcal{L}_B &= \partial_a \left[C^a (\mathcal{L}_0 + B_0 A_0 + B_1 A_1) + i \bar{C}_1 C^a (\partial_0 C^1 + \partial_1 C^0) A_0 \right. \\ &+ i \bar{C}_0 C^b \partial_b (C^a A_0) + i \bar{C}_0 C^a (\partial_0 C^1 + \partial_1 C^0) A_1 + i \bar{C}_1 C^b \partial_b (C^a A_1) \\ &\left. + i \bar{C}_0 C^a (\partial_0 C^1 - \partial_1 C^0) A_2 + i \bar{C}_1 C^a (\partial_0 C^0 - \partial_1 C^1) A_2 \right].\end{aligned}\tag{17}$$

In exactly similar fashion, the anti-BRST transformation acting on the anti-BRST invariant Lagrangian density $\mathcal{L}_{\bar{B}}$ produces the following explicit transformation:

$$\begin{aligned}\bar{s}_B \mathcal{L}_{\bar{B}} &= \partial_a \left[\bar{C}^a (\mathcal{L}_0 - \bar{B}_0 A_0 - \bar{B}_1 A_1) - i C_1 \bar{C}^a (\partial_0 \bar{C}^1 + \partial_1 \bar{C}^0) A_0 \right. \\ &- i C_0 \bar{C}^b \partial_b (\bar{C}^a A_0) - i C_0 \bar{C}^a (\partial_0 \bar{C}^1 + \partial_1 \bar{C}^0) A_1 - i C_1 \bar{C}^b \partial_b (\bar{C}^a A_1) \\ &\left. - i C_0 \bar{C}^a (\partial_0 \bar{C}^1 - \partial_1 \bar{C}^0) A_2 - i C_1 \bar{C}^a (\partial_0 \bar{C}^0 - \partial_1 \bar{C}^1) A_2 \right].\end{aligned}\tag{18}$$

A close and careful look at (17) and (18) shows that we can obtain (18) from (17) provided we make the replacements: $B_0 \rightarrow \bar{B}_0$, $B_1 \rightarrow \bar{B}_1$, $A_0 \rightarrow -A_0$, $A_1 \rightarrow -A_1$, $A_2 \rightarrow -A_2$, $C_0 \leftrightarrow \bar{C}_0$, $C_1 \leftrightarrow \bar{C}_1$. Now it is obvious that, in the flat limit $A_0 = A_1 = 0$, $A_2 = 1$ of the full

Lagrangian densities \mathcal{L}_B and $\mathcal{L}_{\bar{B}}$, we obtain the following BRST and anti-BRST symmetry invariances for the Lagrangian densities $\mathcal{L}_B^{(0)}$ and $\mathcal{L}_{\bar{B}}^{(0)}$, namely;

$$s_B \mathcal{L}_B^{(0)} = \partial_a \left[C^a (\mathcal{L}_0) + i \bar{C}_0 C^a (\partial_0 C^1 - \partial_1 C^0) + i \bar{C}_1 C^a (\partial_0 C^0 - \partial_1 C^1) \right], \quad (19)$$

$$\bar{s}_B \mathcal{L}_{\bar{B}}^{(0)} = \partial_a \left[\bar{C}^a (\mathcal{L}_0) - i C_0 \bar{C}^a (\partial_0 \bar{C}^1 - \partial_1 \bar{C}^0) - i C_1 \bar{C}^a (\partial_0 \bar{C}^0 - \partial_1 \bar{C}^1) \right]. \quad (20)$$

The total derivatives in (17), (18), (19) and (20) establish that the (anti-)BRST transformations (12), (10), (9), (6) and (5) are the symmetries of the action integrals $S = \int d^2\xi \mathcal{L}_B$, $S = \int d^2\xi \mathcal{L}_{\bar{B}}$, $S = \int d^2\xi \mathcal{L}_B^{(0)}$ and $S = \int d^2\xi \mathcal{L}_{\bar{B}}^{(0)}$ provided we use the proper boundary conditions on the fields (and their derivatives) of the theory at $\sigma = 0$ and $\sigma = \pi$ [6].

5 Conserved Charges: Continuous Symmetries

The BRST charge Q_B , that has been computed in [6]. is in the flat limit ($A_0 = A_1 = 0, A_2 = 1$) where the Lagrangian density $\mathcal{L}_B^{(0)}$ (cf. Eqn. (8)) plays a pivotal role. First of all, we note that the Lagrangian densities $\mathcal{L}_B^{(0)}$ and $\mathcal{L}_{\bar{B}}^{(0)}$ (cf. Eqns. (8) and (16)) respect the global ghost-scale symmetry transformations

$$C_0 \rightarrow e^\Omega C_0, \quad \bar{C}_0 \rightarrow e^{-\Omega} \bar{C}_0, \quad C_1 \rightarrow e^\Omega C_1, \quad \bar{C}_1 \rightarrow e^{-\Omega} \bar{C}_1, \quad (21)$$

where Ω is a global scale transformation parameter. For the sake of brevity, we set $\Omega = 1$ so that the infinitesimal version (s_g) of the above global scale symmetry transformation reduces to the following transformations on the (anti-)ghost fields, namely;

$$s_g C_0 = C_0, \quad s_g \bar{C}_0 = -\bar{C}_0, \quad s_g C_1 = C_1, \quad s_g \bar{C}_1 = -\bar{C}_1. \quad (22)$$

Here the subscript g denotes the infinitesimal ghost scale transformations. The ghost charges, computed from $\mathcal{L}_B^{(0)}$ and $\mathcal{L}_{\bar{B}}^{(0)}$, are as follows

$$\begin{aligned} Q_g &= \int_0^\pi d\sigma J_g^{(0)} \equiv -i \int_0^\pi d\sigma (\bar{C}_0 C_1 - \bar{C}_1 C_0), \\ \bar{Q}_g &= \int_0^\pi d\sigma \bar{J}_g^{(0)} \equiv -i \int_0^\pi d\sigma (\bar{C}_1 C_0 - \bar{C}_0 C_1), \end{aligned} \quad (23)$$

where $J_g^{(0)}$ and $\bar{J}_g^{(0)}$ are the zeroth component of the Noether conserved currents (corresponding to the infinitesimal ghost transformations (22)) that have been derived from $\mathcal{L}_B^{(0)}$ and $\mathcal{L}_{\bar{B}}^{(0)}$, respectively. However, the above charges are *not* independent of each-other. Rather, they differ by a sign factor only (i.e. $Q_g = -\bar{Q}_g$). Using the following Euler-Lagrange equations of motion that emerge out from $\mathcal{L}_B^{(0)}$, namely;

$$\begin{aligned} \square X^\mu &= 0, \quad A_0 = A_1 = B_0 = B_1 = 0, \quad A_2 = 1, E = 0, \quad \partial_0 \bar{C}^0 + \partial_1 \bar{C}^1 = 0 \\ \partial_0 C^0 - \partial_1 C^1 &= 0, \quad \partial_0 C^1 - \partial_1 C^0 = 0, \quad \partial_0 \bar{C}^1 + \partial_1 \bar{C}^0 = 0, \end{aligned} \quad (24)$$

we observe that $\dot{Q}_g = i \int_0^\pi \frac{\partial}{\partial \sigma} [\bar{C}_1 C_0 - \bar{C}_0 C_1] = 0$ due to the boundary conditions. This shows that the ghost charge is conserved (i.e. $\dot{Q}_g = 0$).

We now concentrate on the derivation of the BRST charge Q_B and anti-BRST charge \bar{Q}_B from the Lagrangian densities $\mathcal{L}_B^{(0)}$ and $\mathcal{L}_{\bar{B}}^{(0)}$, respectively. Taking into account the basic concepts behind the Noether theorem, we note that $Q_B = \int_0^\pi d\sigma J_B^{(0)}$, $\bar{Q}_B = \int_0^\pi d\sigma \bar{J}_B^{(0)}$ where $J_B^{(0)}$ and $\bar{J}_B^{(0)}$ are the zeroth components of the Noether conserved currents (corresponding to the BRST and anti-BRST symmetry transformations) computed from the Lagrangian densities $\mathcal{L}_B^{(0)}$ and $\mathcal{L}_{\bar{B}}^{(0)}$, respectively. The explicit expressions for these currents, derived from the above Lagrangian densities, are

$$\begin{aligned} J_B^{(0)} &= (s_B X^\mu) \frac{\partial \mathcal{L}_B^{(0)}}{\partial (\partial_0 X^\mu)} + (s_B C^0) \frac{\partial \mathcal{L}_B^{(0)}}{\partial (\partial_0 C^0)} + (s_B C^1) \frac{\partial \mathcal{L}_B^{(0)}}{\partial (\partial_0 C^1)} \\ &+ (s_B \bar{C}_0) \frac{\partial \mathcal{L}_B^{(0)}}{\partial (\partial_0 \bar{C}_0)} + (s_B \bar{C}_1) \frac{\partial \mathcal{L}_B^{(0)}}{\partial (\partial_0 \bar{C}_1)} - X^0, \end{aligned} \quad (25)$$

$$\begin{aligned} \bar{J}_B^{(0)} &= (\bar{s}_B X^\mu) \frac{\partial \mathcal{L}_{\bar{B}}^{(0)}}{\partial (\partial_0 X^\mu)} + (\bar{s}_B C^0) \frac{\partial \mathcal{L}_{\bar{B}}^{(0)}}{\partial (\partial_0 C^0)} + (\bar{s}_B C^1) \frac{\partial \mathcal{L}_{\bar{B}}^{(0)}}{\partial (\partial_0 C^1)} \\ &+ (\bar{s}_B \bar{C}_0) \frac{\partial \mathcal{L}_{\bar{B}}^{(0)}}{\partial (\partial_0 \bar{C}_0)} + (\bar{s}_B \bar{C}_1) \frac{\partial \mathcal{L}_{\bar{B}}^{(0)}}{\partial (\partial_0 \bar{C}_1)} - Y^0, \end{aligned} \quad (26)$$

where the explicit expressions for X^0 and Y^0 are:

$$\begin{aligned} X^0 &= C^0 \mathcal{L}_0 + i \bar{C}_0 C^0 (\partial_0 C^1 - \partial_1 C^0) + i \bar{C}_1 C^0 (\partial_0 C^0 - \partial_1 C^1), \\ Y^0 &= \bar{C}^0 \mathcal{L}_0 - i C_0 \bar{C}^0 (\partial_0 \bar{C}^1 - \partial_1 \bar{C}^0) - i C_1 \bar{C}^0 (\partial_0 \bar{C}^0 - \partial_1 \bar{C}^1). \end{aligned} \quad (27)$$

The above expressions are derived from the equations (19) and (20) which are nothing but the zeroth components of the expressions that have been quoted in the square brackets. Finally, we obtain the following expressions for the conserved BRST and anti-BRST charges (Q_B and \bar{Q}_B) from the Lagrangian densities $\mathcal{L}_B^{(0)}$ and $\mathcal{L}_{\bar{B}}^{(0)}$, namely;

$$\begin{aligned} Q_B &= - \int_0^\pi d\sigma \left[\frac{C^0}{2\kappa} (\partial_0 X^\mu \partial_0 X_\mu + \partial_1 X^\mu \partial_1 X_\mu) + \frac{C^1}{2\kappa} (\partial_0 X^\mu \partial_1 X_\mu + \partial_1 X^\mu \partial_0 X_\mu) \right. \\ &+ i \bar{C}_0 (C^a \partial_a C^1) + i \bar{C}_1 (C^a \partial_a C^0) \Big], \end{aligned} \quad (28)$$

$$\begin{aligned} \bar{Q}_B &= - \int_0^\pi d\sigma \left[\frac{\bar{C}^0}{2\kappa} (\partial_0 X^\mu \partial_0 X_\mu + \partial_1 X^\mu \partial_1 X_\mu) + \frac{\bar{C}^1}{2\kappa} (\partial_0 X^\mu \partial_1 X_\mu + \partial_1 X^\mu \partial_0 X_\mu) \right. \\ &+ i C_0 (\bar{C}^a \partial_a \bar{C}^1) + i C_1 (\bar{C}^a \partial_a \bar{C}^0) \Big], \end{aligned} \quad (29)$$

where we have used the Euler-Lagrange (EL) equations of motion (EoM) (24) derived from the Lagrangian density $\mathcal{L}_B^{(0)}$ and the following EL-EoM that emerge out from the Lagrangian density $\mathcal{L}_{\bar{B}}^{(0)}$, namely;

$$\begin{aligned} \square X^\mu &= 0, & A_0 &= A_1 = A_2 - 1 = \bar{B}_0 = \bar{B}_1 = E = 0, & \partial_0 \bar{C}^0 - \partial_1 \bar{C}^1 &= 0, \\ \partial_0 \bar{C}^1 - \partial_1 \bar{C}^0 &= 0, & \partial_0 C^1 + \partial_1 \bar{C}^0 &= 0, & \partial_0 \bar{C}^0 + \partial_1 \bar{C}^1 &= 0. \end{aligned} \quad (30)$$

In fact, a close and careful look at the EL-EoM (24) and (30) establishes the fact that $X^0 = Y^0 = 0$ on the on-shell (because we substitute the EL-EoM into them).

The above charges Q_B and \bar{Q}_B are conserved. This can be checked by exploiting the strength of the EL-EoM (24) and (30) while we take into account the direct “time” derivative of the above charges, namely;

$$\begin{aligned}\dot{Q}_B = & - \int_0^\pi d\sigma \frac{\partial}{\partial \sigma} \left[\frac{C^0}{2\kappa} (\partial_0 X^\mu \partial_0 X_\mu + \partial_1 X^\mu \partial_1 X_\mu) + \frac{C^1}{2\kappa} (\partial_0 X^\mu \partial_1 X_\mu + \partial_1 X^\mu \partial_0 X_\mu) \right. \\ & \left. + i \bar{C}_0 (C^a \partial_a C^0) + i \bar{C}_1 (C^a \partial_a C^1) \right],\end{aligned}\quad (31)$$

$$\begin{aligned}\dot{\bar{Q}}_B = & - \int_0^\pi d\sigma \frac{\partial}{\partial \sigma} \left[\frac{\bar{C}^0}{2\kappa} (\partial_0 X^\mu \partial_0 X_\mu + \partial_1 X^\mu \partial_1 X_\mu) + \frac{\bar{C}^1}{2\kappa} (\partial_0 X^\mu \partial_1 X_\mu + \partial_1 X^\mu \partial_0 X_\mu) \right. \\ & \left. + i C_0 (\bar{C}^a \partial_a \bar{C}^0) + i C_1 (\bar{C}^a \partial_a \bar{C}^1) \right].\end{aligned}\quad (32)$$

The above expressions demonstrate that the BRST and anti-BRST charges are conserved when we use the boundary conditions at $\sigma = 0$ and $\sigma = \pi$ on the appropriate fields and their derivatives (see, e.g. [6] for details). Thus, we have noted that there are *three* conserved charges (which correspond to *three* continuous symmetries that are present) in the theory. One can check, in a straightforward manner, that the ghost charge obeys the standard algebra with the BRST and anti-BRST charges. This can be checked in a simple manner by computing the left hand side of the following from (22), (28) and (29), namely;

$$s_g Q_g = -i [Q_g, Q_g] = 0, \quad s_g Q_B = -i [Q_B, Q_g] = Q_B, \quad s_g \bar{Q}_B = -i [Q_g, \bar{Q}_B] = -\bar{Q}_B, \quad (33)$$

which demonstrates that we have: $i [Q_g, Q_B] = +Q_B$, and $i [Q_g, \bar{Q}_B] = -\bar{Q}_B$. However, the proof of nilpotency of the BRST and anti-BRST charges requires very careful computations at the *quantum* level where the normal mode expansions of the fields of our theory play very important roles. In the paper by Kato and Ogawa [6], this exercise has been performed and it turns out that the nilpotency of the BRST charge is true *only* when $D = 26$ and $\alpha_0 = 1$. It is obvious that we shall get the same result if we check the nilpotency of the anti-BRST charge at the quantum level with the proper boundary conditions.

6 Conclusions

In our present investigation, we have been able to derive the *proper* anti-BRST symmetry transformations corresponding to the BRST transformations (that have been shown to be present for the standard bosonic string theory [6]). The BRST and anti-BRST symmetry transformations are proved to be off-shell nilpotent of order two. However, these symmetries are found to be absolutely anticommuting *only* on a hypersurface that is characterized by the 2D field equations (11). These latter equations are nothing but the CF-type restrictions which are the hallmark of the *quantum* diffeomorphism/gauge invariant theories when these theories are discussed within the framework of BRST formalism. In fact, it is the existence

of the CF-type restrictions that primarily imply that the BRST and anti-BRST symmetries (and the corresponding charges) have their own identities. In the language of mathematics, they are linearly independent of each-other on the hypersurface that is defined by the CF-type restrictions (11) in the 2D spacetime manifold.

We have derived, in our present endeavor, the explicit forms of BRST and anti-BRST invariant Lagrangian densities and we have demonstrated *clearly* their transformation properties under the BRST and anti-BRST symmetry transformations. Using the Noether theorem, we have computed the conserved BRST, anti-BRST and ghost charges of the theory in the *flat* limit. In fact, in the latter limit, the BRST charge has *also* been derived by Kato and Ogawa [6]. We have shown that the standard algebra is obeyed between the ghost charge and BRST charge (as well as the ghost charge and anti-BRST charge). The nilpotency ($Q_B^2 = 0, \bar{Q}_B^2 = 0$) of the BRST (Q_B) and anti-BRST (\bar{Q}_B) charges has not been derived in our present investigation as this requires the normal mode expansion of the fields and their substitution in the expressions for Q_B and \bar{Q}_B . In fact, the requirement of the nilpotency of the BRST charge has led to the derivation of $D = 26$ and $\alpha_0 = 1$ where D is the dimensionality of the target spacetime manifold and α_0 is the intercept in the Regge trajectory that is generated due to the concept of strings (see, e.g. [6] for details).

We would like to comment a bit on the boundary conditions that are to be imposed on the fields (and the derivatives on them) in our present theory when we demand the BRST as well as anti-BRST invariance of the Lagrangian densities (7) and (15). For the BRST invariance of the theory, the boundary conditions that have been obtained in the work by Kato and Ogawa [6] are: $\partial_1 X^\mu = 0, \bar{C}_0 = 0, C^1 = 0$ at $\sigma = 0$ and $\sigma = \pi$. The BRST invariance of the boundary condition $C^1 = 0$ (at $\sigma = 0$ and $\sigma = \pi$) leads to the further boundary condition as: $\partial_0 C^1 = 0$ at $\sigma = 0$ and $\sigma = \pi$. The anti-BRST invariance, in exactly similar manner, would lead to the boundary conditions $\partial_1 X^\mu = 0, C^0 = 0, \bar{C}_1 = 0$ at $\sigma = 0$ and $\sigma = \pi$. The anti-BRST invariance of the condition $\bar{C}_1 = 0$ at $\sigma = 0$ and $\sigma = \pi$ implies that $\partial_0 \bar{C}_1 = 0$ (at $\sigma = 0$ and $\sigma = \pi$). Thus, the normal mode expansions of the fields: $X^\mu(\tau, \sigma), C^0(\tau, \sigma), C^1(\tau, \sigma), \bar{C}_0(\tau, \sigma), \bar{C}_1(\tau, \sigma)$ can be found in the same manner as has been obtained in the work by Kato and Ogawa [6]. We have to be just careful that for the anti-BRST invariance, the mode expansions in the ghost sector should be such that the expansions are exchanged, namely; $C_a \leftrightarrow \bar{C}_a$. The requirements of the nilpotency of Q_B and \bar{Q}_B would obviously produce the results $D = 26$ and $\alpha_0 = 1$.

We would like to mention that the BRST and anti-BRST invariant Lagrangian densities (7) and (15) have been derived in a straightforward manner by utilizing the gauge-fixing $A_0 = A_1 = 0$ and the (anti-)ghost fields (cf. Eqns. (4) and (14)). However, if we compute the Lagrangian densities in the Curci-Ferrari gauge [12,13] that would give due respect to the CF-type conditions that have been derived in (11). We wish to devote time on the computation of the coupled Lagrangian densities (like 4D non-Abelian gauge theory [10-13]) which produce the CF-type condition as the equations of motion. Furthermore, the coupled Lagrangian densities should respect *both* the BRST and anti-BRST symmetry transformations on the hypersurface where CF-type restrictions (11) are satisfied. At present, we are working in this direction and our results would be reported elsewhere.

In a very recent work [7], the superfield approach to derive the proper (anti-)BRST symmetry transformations for any general diffeomorphism theory has been developed (corresponding to its diffeomorphism symmetry invariance). It would be very nice future en-

deavor for us to apply the theoretical arsenals of this superfield formalism [7] to our present bosonic string model which is *also* a diffeomorphism invariant theory. In fact, we hope that this superfield formalism would be able to shed more light on the geometrical origin and interpretation of the (anti-)BRST symmetries and the CF-type restrictions (11) which we have obtained in our present theory. In our earlier work [14], we have established the geometrical origin of CF-type restriction and its connection with gerbes. It would be a challenging future endeavor for us to establish the connection of the CF-type restrictions (11) with the concept of gerbes. We are presently involved with this problem and we shall be able to report about our progress in our future publications [15].

Acknowledgements

Fruitful conversations with L. Bonora (SISSA, Trieste, Italy) are gratefully acknowledged as he has been a constant source of inspiration and encouragement during the completion of this work. Thanks are also due to T. Bhanja for his computational skills and help in reading of the present manuscript.

Appendix A: On the Nilpotency Property $s_B^2 \tilde{g}^{ab} = 0$

We briefly sketch here a few essentials steps that are needed in the proof of $s_B^2 \tilde{g}^{ab} = 0$. In this connection, we observe the following:

$$s_B^2 \tilde{g}^{ab} = s_B [(\partial_m C^m) \tilde{g}^{ab}] + s_B [C^m \partial_m \tilde{g}^{ab}] - s_B [(\partial_m C^a) \tilde{g}^{mb}] - s_B [(\partial_m C^b) \tilde{g}^{am}]. \quad (34)$$

The first term, after the application of the BRST transformations, looks in its full glory as

$$\begin{aligned} & (\partial_m C^n) (\partial_n C^m) \tilde{g}^{ab} + C^m (\partial_m \partial_n C^n) \tilde{g}^{ab} - (\partial_m C^m) (\partial_n C^n) \tilde{g}^{ab} - (\partial_m C^m) C^n (\partial_n \tilde{g}^{ab}) \\ & + (\partial_m C^m) (\partial_n C^a) \tilde{g}^{nb} + (\partial_m C^m) (\partial_n C^b) \tilde{g}^{an}. \end{aligned} \quad (35)$$

In exactly similar fashion, the second term turns out to be

$$\begin{aligned} & C^m (\partial_n C^m) (\partial_m \tilde{g}^{ab}) - C^m (\partial_m \partial_n C^n) \tilde{g}^{ab} - C^m (\partial_n C^n) (\partial_m \tilde{g}^{ab}) - C^m (\partial_m C^n) (\partial_n \tilde{g}^{ab}) \\ & + C^n (\partial_n C^a) (\partial_m \tilde{g}^{mb}) + C^n (\partial_n C^b) (\partial_m \tilde{g}^{an}) + C^m (\partial_m \partial_n C^a) \tilde{g}^{mb}, \end{aligned} \quad (36)$$

where we have taken into account the fact that $C^m C^n (\partial_m \partial_n \tilde{g}^{ab}) = 0$. The third term, after the application of the BRST transformations (5), (6) and (12), looks in the following exact mathematical form

$$\begin{aligned} & -(\partial_m C^n) (\partial_n C^a) \tilde{g}^{mb} - C^m (\partial_m \partial_n C^a) \tilde{g}^{mb} + (\partial_m C^a) (\partial_n C^n) \tilde{g}^{mb} \\ & + (\partial_m C^a) C^m (\partial_n \tilde{g}^{mb}) - (\partial_m C^a) (\partial_n C^m) \tilde{g}^{nb} - (\partial_m C^a) (\partial_n C^b) \tilde{g}^{mn}. \end{aligned} \quad (37)$$

Finally, the fourth term can be explicitly expressed, after the application of BRST transformations (5), (6) and (12), as

$$\begin{aligned} & -(\partial_m C^n)(\partial_n C^b) \tilde{g}^{am} - C^m (\partial_m \partial_n C^b) \tilde{g}^{am} + (\partial_m C^b)(\partial_n C^m) \tilde{g}^{am} + (\partial_m C^b) C^m (\partial_n \tilde{g}^{am}) \\ & -(\partial_m C^b)(\partial_n C^m) \tilde{g}^{an} - (\partial_m C^b)(\partial_n C^a) \tilde{g}^{mn}. \end{aligned} \quad (38)$$

It is evident that the following terms from (35), (36), (37) and (38), namely;

$$\begin{aligned} & C^n (\partial_m \partial_n C^m) \tilde{g}^{ab} - C^m (\partial_m \partial_n C^m) \tilde{g}^{ab} + C^m (\partial_m \partial_n C^a) \tilde{g}^{mb} \\ & C^m (\partial_m \partial_n C^b) \tilde{g}^{an} - C^m (\partial_m \partial_n C^a) \tilde{g}^{mb} - C^m (\partial_m \partial_n C^b) \tilde{g}^{an}, \end{aligned} \quad (39)$$

cancel out with one-another. Furthermore, the following terms from (37) and (38),

$$-(\partial_m C^a)(\partial_n C^b) \tilde{g}^{mn} - (\partial_m C^b)(\partial_n C^a) \tilde{g}^{mn}, \quad (40)$$

cancel out with each-other because of the antisymmetric nature ($C^a C^b + C^b C^a = 0$) of the ghost fields (C^a) and the symmetric nature ($\tilde{g}^{mn} = \tilde{g}^{nm}$) of the metric tensor \tilde{g}^{mn} . Rest of the terms also cancel out by taking the help of the exchange of dummy indices $m \leftrightarrow n$ and the antisymmetric nature of the ghost fields. Finally, we find that the following terms, from the sum of (35), (36), (37) and (38), remain left-out at the end, namely;

$$\left[(\partial_n C^m)(\partial_m C^n) - (\partial_m C^m)(\partial_n C^n) \right] \tilde{g}^{ab}. \quad (41)$$

The terms in the square bracket cancel with each-other when we take the sum over $m, n = 0, 1$. This establishes the nilpotency ($s_B^2 \tilde{g}^{ab} = 0$) of the metric tensor \tilde{g}^{ab} .

Appendix B: On the BRST Symmetry Invariance of \mathcal{L}_B

We collect here all the terms that are generated due to the application of BRST symmetry transformations (s_B) on \mathcal{L}_B (cf. Eqn. (7)). It is straightforward to note that $s_B \mathcal{L}_0 = \partial_a (C^a \mathcal{L}_0)$. We assemble, first of all, the terms that contain B_0 and B_1 fields due to the application of s_B on *all* the terms that are present in \mathcal{L}_B . These terms with B_0 field are

$$\begin{aligned} & C^a (\partial_a B_0) A_0 + B_0 C^a (\partial_a A_0) - B_0 (\partial_0 C^1 + \partial_1 C^0) A_1 \\ & - B_0 (\partial_0 C^1 - \partial_1 C^0) A_2 - B_0 (\partial_1 C^0) A_2 + B_0 (\partial_0 C^1) A_2 \\ & + B_0 (\partial_1 C^0) A_1 + B_0 (\partial_0 C^1) A_1 + B_0 (\partial_a C^a) A_0. \end{aligned} \quad (42)$$

Similarly, the terms containing B_1 fields are as follows:

$$\begin{aligned} & B_1 C^a (\partial_a A_1) - B_1 (\partial_a C^a) A_2 + B_1 (\partial_1 C^1) A_2 - B_1 (\partial_1 C^0) A_0 \\ & + B_1 (\partial_a C^a) A_1 - B_1 (\partial_0 C^1) A_0 - B_1 (\partial_1 C^1) A_2 + B_1 (\partial_0 C^0) A_2 \\ & + B_1 (\partial_0 C^1) A_0 + B_1 (\partial_1 C^0) A_0 + C^a (\partial_a B_1) A_1. \end{aligned} \quad (43)$$

It is clear that if we sum these terms (i.e. (42) and (43)) carefully with $s_B \mathcal{L}_0 = \partial_a (C^a \mathcal{L}_0)$, they lead to the sum of the following total derivative:

$$\partial_a \left[C^a (\mathcal{L}_0 + B_0 A_0 + B_1 A_1) \right]. \quad (44)$$

Thus far, we have obtained the total derivative from the original Lagrangian density (1) and terms that contain necessarily the Nakanishi-Lautrup fields B_0 and B_1 .

We now collect the terms that incorporate A_2 after the application of s_B on \mathcal{L}_B (cf. Eqn. (7)). These are listed as follows:

$$\begin{aligned} & i \bar{C}_1 (\partial_1 C^1) C^a (\partial_a A_2) - i C^a (\partial_a \bar{C}_0) (\partial_0 C^1 - \partial_1 C^0) A_2 - i \bar{C}_1 (\partial_0 C^0) C^a \partial_a A_2 \\ & + i \bar{C}_0 (\partial_1 C^0) C^a (\partial_a A_2) - i \bar{C}_0 (\partial_0 C^1) C^a (\partial_a A_2) + i \bar{C}_1 (\partial_a C^a) (\partial_0 C^0 - \partial_1 C^1) A_2 \\ & - i C^a (\partial_a \bar{C}_1) (\partial_0 C^0 - \partial_1 C^1) A_2 + i \bar{C}_0 (\partial_a C^a) (\partial_0 C^1 - \partial_1 C^0) A_2 - i \bar{C}_1 C^a (\partial_a \partial_1 C^1) A_2 \\ & + i \bar{C}_1 (\partial_0 C^1) (\partial_0 C^1 - \partial_1 C^0) A_2 + i \bar{C}_1 (\partial_1 C^0) (\partial_0 C^1 - \partial_1 C^0) A_2 - i \bar{C}_1 (\partial_1 C^a) (\partial_a C^1) A_2 \\ & - i \bar{C}_1 (\partial_0 C^a) (\partial_a C^0) A_2 + i \bar{C}_1 C^a (\partial_a \partial_0 C^0) A_2 + i \bar{C}_0 (\partial_0 C^1) (\partial_0 C^0 - \partial_1 C^1) A_2 \\ & - i \bar{C}_0 (\partial_1 C^a) (\partial_a C^0) A_2 - i \bar{C}_0 C^a (\partial_a \partial_1 C^0) A_2 + i \bar{C}_0 (\partial_0 C^a) (\partial_a C^1) A_2 \\ & + i \bar{C}_0 C^a (\partial_a \partial_0 C^1) A_2 + i \bar{C}_0 (\partial_1 C^0) (\partial_0 C^0 - \partial_1 C^1) A_2. \end{aligned} \quad (45)$$

It is very interesting to note that all these terms, after many surprising cancellations, sum-up to yield a total derivative as:

$$\partial_a \left[i \bar{C}_0 C^a (\partial_0 C^1 - \partial_1 C^0) A_2 + i \bar{C}_1 C^a (\partial_0 C^0 - \partial_1 C^1) A_2 \right]. \quad (46)$$

We now concentrate on all the terms that contain A_0 which emerge out from the application of s_B on the relevant terms of the Lagrangian density \mathcal{L}_B . These terms are

$$\begin{aligned} & i \bar{C}_1 (\partial_0 C^0) (\partial_1 C^0 - \partial_0 C^1) A_0 - i \bar{C}_1 (\partial_1 C^1) (\partial_1 C^0 - \partial_0 C^1) A_0 \\ & - i \bar{C}_0 (\partial_1 C^0) (\partial_1 C^0 - \partial_0 C^1) A_0 + i \bar{C}_0 (\partial_0 C^1) (\partial_1 C^0 - \partial_0 C^1) A_0 \\ & + i \bar{C}_0 (\partial_1 C^0) (\partial_1 C^0 + \partial_0 C^1) A_0 + i \bar{C}_0 (\partial_0 C^1) (\partial_1 C^0 + \partial_0 C^1) A_0 \\ & + i \bar{C}_1 C^a (\partial_1 \partial_a C^0) A_0 - i \bar{C}_1 (\partial_1 C^0) C^a \partial_a A_0 + i \bar{C}_1 (\partial_0 C^a) (\partial_a C^1) A_0 \\ & + i \bar{C}_1 C^a (\partial_0 \partial_a C^1) A_0 - i \bar{C}_1 (\partial_0 C^1) C^a \partial_a A_0 + i \bar{C}_0 (\partial_a C^b) (\partial_b C^a) A_0 \\ & + i \bar{C}_0 C^b (\partial_b \partial_a C^a) A_0 - i \bar{C}_0 (\partial_a C^a) C^b \partial_b A_0 + i \bar{C}_b (\partial_b C^a) (\partial_a \bar{C}_0) A_0 \\ & + i C^a (\partial_a \bar{C}_0) C^b \partial_b A_0 + i \bar{C}_1 (\partial_1 C^a) (\partial_a C^0) A_0. \end{aligned} \quad (47)$$

It is amazing to find out that the sum of the above terms, after some miraculous cancellations, yields a total derivative as:

$$\partial_a [i \bar{C}_1 C^a (\partial_0 C^1 + \partial_1 C^0) A_0 + i \bar{C}_0 C^b \partial_b (C^a A_0)]. \quad (48)$$

Finally, we focus on the terms that necessarily incorporate A_1 field after the application of

the BRST transformation s_B on the Lagrangian density (7). These terms are

$$\begin{aligned}
& i \bar{C}_0 (\partial_a C^a) (\partial_0 C^1 + \partial_1 C^0) A_1 + i \bar{C}_1 C^b \partial_b \partial_a C^a A_1 - i \bar{C}_1 (\partial_a C^a) C^b \partial_b A_1 \\
& + i C^b \partial_b C^a (\partial_a \bar{C}_1) A_1 + i C^b (\partial_b \bar{C}_1) C^a \partial_a A_1 - i \bar{C}_0 (\partial_0 C^1) C^a \partial_a A_1 \\
& + i \bar{C}_1 (\partial_a C^b) C^a \partial_a A_1 - i \bar{C}_0 (\partial_1 C^0) C^a \partial_a A_1 + i \bar{C}_0 C^a (\partial_0 \partial_a C^1) A_1 \\
& - i \bar{C}_1 (\partial_1 C^1) (\partial_0 C^0 - \partial_1 C^1) A_1 + i \bar{C}_1 (\partial_0 C^0) (\partial_0 C^0 - \partial_1 C^1) A_1 \\
& - i \bar{C}_0 (\partial_1 C^0) (\partial_0 C^0 - \partial_1 C^1) A_1 + i \bar{C}_0 (\partial_0 C^1) (\partial_0 C^0 - \partial_1 C^1) A_1 \\
& - i \bar{C}_0 (\partial_1 C^a) (\partial_a C^0) A_1 + i \bar{C}_0 C^a (\partial_1 \partial_a C^0) A_1 + i \bar{C}_0 (\partial_0 C^a) (\partial_a C^1) A_1 \\
& + i \bar{C}_1 (\partial_0 C^1) (\partial_0 C^1 + \partial_1 C^0) A_1 + i \bar{C}_1 (\partial_1 C^0) (\partial_0 C^1 + \partial_1 C^0) A_1 \\
& - i C^a (\partial_a \bar{C}_0) (\partial_0 C^1 + \partial_1 C^0) A_1.
\end{aligned} \tag{49}$$

The above terms add-up to yield a total derivative term as:

$$\partial_a [i \bar{C}_0 C^a (\partial_0 C^1 + \partial_1 C^0) A_1 + i \bar{C}_1 C^b \partial_b (C^a A_1)]. \tag{50}$$

It is interesting to point out that the terms with A_0 and that of A_1 sum-up to yield exactly similar types of result in the total derivative where $A_0 \leftrightarrow A_1$, $\bar{C}_0 \leftrightarrow \bar{C}_1$. It is clear that the application of s_B on \mathcal{L}_B produces the total derivative term which is the sum of (44), (46), (48) and (50). Thus, the BRST transformations s_B is a symmetry of the action.

Appendix C: On the Anti-BRST Symmetry Invariance of $\mathcal{L}_{\bar{B}}$

We collect here the terms that are generated after the application of the anti-BRST symmetry transformations \bar{s}_B on $\mathcal{L}_{\bar{B}}$ (cf. Eqn. (15)). It can be readily checked that $\bar{s}_B \mathcal{L}_0 = \partial_a (\bar{C}^a \mathcal{L}_0)$. In addition to it, we have the following terms that contain the auxiliary field \bar{B}_0 after the application of \bar{s}_B on $\mathcal{L}_{\bar{B}}$, namely;

$$\begin{aligned}
& -\bar{C}^a (\partial_a \bar{B}_0) A_0 - \bar{B}_0 \bar{C}^a (\partial_a A_0) + \bar{B}_0 (\partial_0 \bar{C}^1 + \partial_1 \bar{C}^0) A_1 \\
& + \bar{B}_0 (\partial_0 \bar{C}^1 - \partial_1 \bar{C}^0) A_2 + \bar{B}_0 (\partial_1 \bar{C}^0) A_2 - \bar{B}_0 (\partial_0 \bar{C}^1) A_2 \\
& - \bar{B}_0 (\partial_1 \bar{C}^0) A_1 - \bar{B}_0 (\partial_0 \bar{C}^1) A_1 - \bar{B}_0 (\partial_a \bar{C}^a) A_0,
\end{aligned} \tag{51}$$

which add-up to yield $\partial_a [-\bar{C}^a \bar{B}_0 A_0]$. Similarly, the following terms containing \bar{B}_1 fields (that are generated after the application of \bar{s}_B on $\mathcal{L}_{\bar{B}}$), namely;

$$\begin{aligned}
& -\bar{B}_1 \bar{C}^a (\partial_a A_1) + \bar{B}_1 (\partial_0 \bar{C}^0) A_2 - \bar{B}_1 (\partial_1 \bar{C}^1) A_2 + \bar{B}_1 (\partial_1 \bar{C}^0) A_0 \\
& - \bar{B}_1 (\partial_a \bar{C}^a) A_1 + \bar{B}_1 (\partial_0 \bar{C}^1) A_0 + \bar{B}_1 (\partial_1 \bar{C}^1) A_2 - \bar{B}_1 (\partial_0 \bar{C}^0) A_2 \\
& - \bar{B}_1 (\partial_0 \bar{C}^1) A_0 - \bar{B}_1 (\partial_1 \bar{C}^0) A_0 - \bar{C}^a (\partial_a \bar{B}_1) A_1,
\end{aligned} \tag{52}$$

sum-up to produce $\partial_a [-\bar{C}^a \bar{B}_1 A_0]$. Thus, it is clear that we have so far the following total derivatives: $\partial_a [\bar{C}^a (\mathcal{L}_0 - \bar{B}_0 A_0 - \bar{B}_1 A_1)]$. Now we focus on the collection of A_0 terms that are generated after the application of anti-BRST transformations \bar{s}_B on $\mathcal{L}_{\bar{B}}$. These are

$$\begin{aligned}
& + i \partial_a (C_0 \bar{C}^a) (\bar{C}^b \partial_b A_0) + i C_1 (\partial_1 \bar{C}^0 + \partial_0 \bar{C}^1) (\bar{C}^b \partial_b A_0) \\
& - \partial_a [i C_0 (\bar{C}^b \partial_b \bar{C}^a)] A_0 - i C_1 \partial_0 (\bar{C}^b \partial_b \bar{C}^1) A_0 - i C_1 \partial_1 (\bar{C}^b \partial_b \bar{C}^0) A_0 \\
& - i \partial_a (C_1 \bar{C}^a) (\partial_1 \bar{C}^0 + \partial_0 \bar{C}^1) A_0 - i C_1 (\partial_0 \bar{C}^0 - \partial_1 \bar{C}^1) (\partial_1 \bar{C}^0 - \partial_0 \bar{C}^1) A_0.
\end{aligned} \tag{53}$$

It will be noted that we have collected here the A_0 terms which look completely different from the corresponding terms in the BRST symmetry invariance (cf. (47)). This is due to the fact we have not written each term separately and independently. However, these terms are actually similar to (47). The above terms add-up to produce the following total derivative terms, namely,

$$\partial_a \left[-i C_1 \bar{C}^a (\partial_0 \bar{C}^1 + \partial_1 \bar{C}^0) A_0 - i C_0 \bar{C}^b \partial_b (\bar{C}^a A_0) \right]. \quad (54)$$

We now concentrate on all the terms that are generated after the application of \bar{s}_B on $\mathcal{L}_{\bar{B}}$ and contain necessarily A_1 field. These are as follows

$$\begin{aligned} & + i \partial_a (C_1 \bar{C}^a) (\bar{C}^b \partial_b A_1) + i C_0 (\partial_1 \bar{C}^0 + \partial_0 \bar{C}^1) (\bar{C}^b \partial_b A_1) \\ & - \partial_a [i C_1 (\bar{C}^b \partial_b \bar{C}^a)] A_1 - i C_0 \partial_0 (\bar{C}^b \partial_b \bar{C}^1) A_1 - i C_0 \partial_1 (\bar{C}^b \partial_b \bar{C}^0) A_1 \\ & - i \partial_a (C_0 \bar{C}^a) (\partial_1 \bar{C}^0 + \partial_0 \bar{C}^1) A_1 - i C_1 (\partial_0 \bar{C}^0 - \partial_1 \bar{C}^1) (\partial_1 \bar{C}^0 - \partial_0 \bar{C}^1) A_1. \end{aligned} \quad (55)$$

The above terms add-up to produce the following total derivative

$$\partial_a \left[-i C_0 \bar{C}^a (\partial_0 \bar{C}^1 + \partial_1 \bar{C}^0) A_1 - i C_1 \bar{C}^b \partial_b (\bar{C}^a A_1) \right]. \quad (56)$$

Finally, we have the following terms that contain necessarily A_2 field after the application of \bar{s}_B on $\mathcal{L}_{\bar{B}}$, namely;

$$\begin{aligned} & - \partial_a [i C_0 \bar{C}^a] (\partial_0 \bar{C}^1 - \partial_1 \bar{C}^0) A_2 - i C_0 (\partial_0 \bar{C}^1 + \partial_1 \bar{C}^0) (\partial_0 \bar{C}^0 - \partial_1 \bar{C}^1) A_2 \\ & + i C_0 (\partial_0 \bar{C}^1 - \partial_1 \bar{C}^0) (\bar{C}^b \partial_b A_2) - i \partial_a (C_1 \bar{C}^a) (\partial_0 \bar{C}^0 - \partial_1 \bar{C}^1) A_2 \\ & + i C_1 (\partial_0 \bar{C}^0 - \partial_1 \bar{C}^1) (\bar{C}^b \partial_b A_2) - i C_0 \partial_0 (\bar{C}^b \partial_b \bar{C}^1) A_2 - i C_1 \partial_0 (\bar{C}^b \partial_b \bar{C}^0) A_2 \\ & + i C_a \partial_1 (\bar{C}^b \partial_b \bar{C}^a) A_2 + 2 i C_1 \partial_0 \bar{C}^1 (\partial_1 \bar{C}^0) A_2. \end{aligned} \quad (57)$$

The above terms produce, after their addition, the following total derivative:

$$\partial_a \left[-i C_0 \bar{C}^a (\partial_0 \bar{C}^1 - \partial_1 \bar{C}^0) A_2 - i C_1 \bar{C}^a (\partial_0 \bar{C}^0 - \partial_1 \bar{C}^1) A_2 \right]. \quad (58)$$

The sum of all the total derivatives, present in this Appendix, sum-up to produce the total derivative that has been quoted in the main body of our text (cf. (18)).

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